Differential Calculus

- **Limits**
  - The number a is called **cluster point** of a sequence a_n, if an inifnte nmber of terms of the seuqnce clusters close to a
  - The number a is called **limit** of the sequence a_n, if there exists $n_0 \in \mathbb{N}$ so that **all** terms a_n with $n > n_0$ cluster close to a
  $$\lim_{n\to\infty} a_n = a \text{ where } a \text{ is finite}$$
  such a sequence is **convergent** and has always a **cluster point**
- **Limit Theorems** hold true if a_n and b_n are convergent
  - $\lim_{n\to\infty}(a_n \pm b_n) = \lim_{n\to\infty}(a_n) \pm \lim_{n\to\infty}(b_n) = a \pm b$
  - $\lim_{n\to\infty}(a_n * b_n) = \lim_{n\to\infty}(a_n) * \lim_{n\to\infty}(b_n) = a * b$
  - $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{\lim_{n\to\infty}(a_n)}{\lim_{n\to\infty}(b_n)} = \frac{a}{b}$ $for\ b_n \neq 0, b \neq 0$
- **Limit of Functions**
  - $\lim_{x\to a} c = c$
  - $\lim_{x\to a} c * f(x) = c * \lim_{x\to a} f(x)$
  - $\lim_{x\to a}\big(f(x) \pm g(x)\big) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
  - $\lim_{x\to a}\big(f(x) * g(x)\big) = \lim_{x\to a} f(x) * \lim_{x\to a} g(x)$
  - $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ $if\ \lim_{x\to a} g(x) \neq 0$
- A **secant** of a curve is a straight line segment which joins any two points on the curve. The slope of the secant measures the **average rate of change**
  $$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$
- A **tangent** is a straight line which touches a curve a t point. The slope of the tangent measures the **instantaneous rate of change** at this point
  $$t(x) = m * x + q \text{ where } m(x) = f'(x)$$
- A **normal** is perpendicular to the tangent
  $$m_t * m_n = -1$$
- The **derivative (differential quotient)** is the slope of the **tangent** to the curve. The derivative of $f(x)$ is written as $f'(x), y'(x), y'\ or\ \frac{df}{dx}(x)$
  It can be found using two different methods
  - **h-method**
  $$\lim_{h\to 0}\frac{f(a+h) - f(a)}{h}$$
  - **x-method**
  $$\lim_{x\to a}\frac{f(x) - f(a)}{x - a}$$
- **Derivative Rules**
  - The Constant Rule
  $$f(x) = c \to f'(x) = 0$$
  - The Sum Rule
  $$f(x) = u(x) + v(x) \to f'(x) = u'(x) + v'(x)$$
  - The Power Rule
  $$f(x) = x^n \to f'(x) = n * x^{n-1}$$
  - The Factor Rule
  $$f(x) = c * u(x) \to f'(x) = c * u'(x)$$